Lec 9: 09/23/2010

Equations of Stellar Evolution (Cont'd):

If the mass of the star M is specified, the equation for stellar structure that we derived last time can be solved to determine various physical parameters as functions of radius. They also determine the total radius R, luminosity L, central temperature T_c, and central pressure P_c.

For the total radius one finds approximate expressions as follows:

$$\boxed{\begin{cases} R \propto M^{0.85} & M \lesssim M_\odot \\ R \propto M^{0.56} & M \gtrsim M_\odot \end{cases}}$$

It is seen that there is a distinct change around $M \simeq M_\odot$. As we will see, the internal structure of the star is quite different based on whether $M < M_\odot$ or $M > M_\odot$. It turns out that the central temperature T_c changes by a factor of 3 for $M_\odot \lesssim M \lesssim 60\,M_\odot$.

The total luminosity of the star L is a strong function of its mass. It can be approximated by the following fitting functions:

$$\frac{L}{L_\odot} \propto \begin{cases} 7^{1.75}\left(\frac{M}{M_\odot}\right)^{3} & M \gtrsim 7\, M_\odot \\ \left(\frac{M}{M_\odot}\right)^{4.8} & 0.4\, M_\odot \lesssim M \lesssim 7\, M_\odot \\ 0.4^{2.85}\left(\frac{M}{M_\odot}\right)^{1.9} & M \lesssim 0.4\, M_\odot \end{cases}$$

Again, the steep dependence for $M \gtrsim M_\odot$ arises because of the strong dependence of the nuclear energy production rate on temperature for stars with $M \gtrsim M_\odot$.

For $M \gtrsim M_\odot$, the approximate scaling is given by:

$$\frac{L}{L_\odot} \simeq \left(\frac{M}{M_\odot}\right)^{3.5} \quad *$$

This relation can be used to estimate the time scale over which the nuclear fuel can sustain the star. During Hydrogen burning $\varepsilon = 6\times 10^{18}\ \frac{erg}{g}$. Assuming that a star ceases to be in a time-independent state (as evolutionary effects become important) when 10% of the original mass is converted to Helium, we can estimate the corresponding time scale as:

$$t_{nuc} \simeq 0.1\,\varepsilon\,\frac{XM}{L} \simeq 10^{10}\left(\frac{M}{M_\odot}\right)\left(\frac{L}{L_\odot}\right)^{-1}\ yr$$

Here $X \simeq 0.74$ is the mass fraction of Hydrogen. After using the expression in equation *, we find:

$$\boxed{t_{nuc} \simeq 10^{10}\left(\frac{M}{M_\odot}\right)^{-2.5}\ yr}$$

The main-sequence lifetime of a star with $M \simeq M_\odot$ is therefore $\sim 10^{10}$ years. More massive stars spend (significantly) less time in the main-sequence phase, which has important theoretical and observational consequences.

Thinking of a star as a blackbody with some effective surface temperature T_e, its luminosity will be given by $L = 4\pi R^2 \sigma T_e^4$

(σ being the Stefan-Boltzmann constant). Normalizing by solar luminosity and radius, we find:

$$\frac{L}{L_\odot} = 8.973 \times 10^{-16} \left(\frac{R}{R_\odot}\right)^2 T_e^4$$

After using the M-R scaling (discussed in Lec. 4) and the expression in equation *, we have:

$$\boxed{T_e \simeq T_{\odot,e} \left(\frac{L}{L_\odot}\right)^\eta \qquad \eta = 0.12 - 0.17}$$

Because T_e and L are measurable (the former from fitting to a black body spectrum, the latter if the distance is known), it is possible to plot the position of the stars in the L-T_e plane, which is called the H-R (Hertzsprung-Russel) diagram.

It is seen from the above expressions that as the mass increases, the main-sequence stars get bigger and brighter (also less dense).

Now let us discuss an important difference in the structure of stars with $M \gtrsim M_\odot$ and those with $M \lesssim M_\odot$. For $M \gtrsim M_\odot$, the central regions of the star are convective and the rest of the star is radiative. The size of the central convective region increases monotonically with M for $M \gtrsim 1.2\, M_\odot$. On the other hand, for $M \lesssim M_\odot$, the central region of the star is radiative, while the outer envelope is convective. The transition from a convective core to a radiative core occurs around $M \simeq 1.2\, M_\odot$. For $M \lesssim 0.3\, M_\odot$, the entire star becomes convective.

The physical reasons for these two phenomena are different, but can be understood simply. Stars with $M \gtrsim M_\odot$ have central temperatures that are higher than $\sim 1.4 \times 10^7$ K. The CNO-cycle will be the dominant mechanism of energy production in these stars. Since this reaction is very sensitive to temperature, energy production will increase rapidly near the center. This results in an increase in ∇_{rad}, and hence energy transport happens in the convective mode near the center. However, ∇_{rad} decreases at larger radii, and the energy transport switches to the radiative mode.

There are two additional factors to convective instability in the stars with $M \gtrsim M_\odot$:

1) When M increases, the central temperature goes up and the relative importance of P_{rad} increases:

$$P = \left(\frac{\rho}{\nu m_H}\right) k_B T + \underbrace{\frac{1}{3} a T^4}_{P_{rad}}$$

(ν: mean molecular weight, $\nu \simeq 1$ for the 100% Hydrogen case $X=1$)

For example, for $M \simeq 50 M_\odot$, we have $P_{rad} \simeq 0.3 P_{tot}$ near the center. As a result, ∇_{ad} decreases by a factor $1 - \frac{P_{rad}}{P_{tot}}$. Lowering ∇_{ad} helps convective instability.

(relative to its ideal gas value $\frac{\gamma - 1}{\gamma}$)

2) The ionization of Hydrogen (as we move from the cooler outer layers towards the hot interior) also plays an important role in lowering ∇_{ad}. The reason being that free electrons contribute to the pressure just like ions, but the dominant contribution to the mass comes from ions. This decreases pressure from ions relative to the total pressure. In fact, intermediate stars can have two convective zones in them where Hydrogen and Helium are being ionized.

Now lets consider stars with $M \lesssim M_\odot$. As M is reduced, the central temperature decreases and so does the relative importance of the CNO-cycle. This causes the size of the convective core to shrink, and it finally disappears around $M \simeq 1.2\ M_\odot$. Around the same value of stellar mass, the outer regions begin to be convective. This has to do with the opacity κ

in the outer region of low mass stars;

$\kappa = \kappa_{es} + \kappa_{ff} + \kappa_{bf}$

The first contribution is due to scattering of photons off free electrons, and $\kappa_{es} = \text{const.}$ (recall that the scattering cross-section σ_T is constant). The last two terms, κ_{ff} and κ_{bf}, are due to free-free and bound-free transitions from photon absorption respectively. They become larger ($\propto T^{-3.5}$) when ionization and recombination of Hydrogen takes place around 10^5 K. Since $\nabla_{rad} \propto \kappa$, an increase in κ in the cooler outer layers results in an increase in ∇_{rad}, and hence switching from radiative to convective mode. As the mass of the star is lowered, the size of the outer convective region increases. For $M \lesssim 0.3\, M_\odot$, the entire star becomes convective.